Experimental Investigation of Undulating Radially Expanding Free Liquid Sheets Using Chromatic Confocal Measurements

Alexander Gyurkovich*, Carsten Mehring
Institute of Mechanical Process Engineering, University of Stuttgart, 70199 Stuttgart, Germany
*Corresponding author: Alexander.Gyurkovich@imvt.uni-stuttgart.de

Abstract
Thin free liquid films are relevant in many technical applications, including spraying technology. In many atomizers liquid films are formed as an intermediate structure in a highly dynamic drop formation process [2] [6] [17]. Capillary waves play an important role in the overall film disintegration process and therefore continue to be the subject of basic theoretical and experimental research. In particular, undulating, radially expanding axisymmetric free liquid sheets were recently investigated experimentally and analytically by Bremond et al. [1], Tirumkudulu and Paramati [16], Paramati et al. [12] and Majumdar and Tirumkudulu [9] [10]. In their higher-order perturbation analysis of radial films without gas-phase interaction, Tirumkudulu and Paramati describe a sinuous-mode film instability attributed to the effect of film thinning (a consequence of the radial geometry) which is not identified as part of the aerodynamic stability analysis presented by Bremond et al. [1]. The present work investigates radially expanding films similar to those described above by using the chromatic confocal measurement technique. Radially expanding sheets are generated by a liquid jet impinging onto the flat end of a circular cylinder whose vertical position can be undulated at a set frequency and amplitude thereby imposing harmonic sinuous disturbances onto the sheets emanating from the impinging surface. Using a chromatic confocal sensor, the local time-dependent vertical displacement of the sheet surface is measured at different radial locations and with a frequency of 4 kHz. After applying a high-pass filter the collected time-dependent data is processed to obtain the amplitude envelope, i.e. the variation of the envelope of the undulating sheet as a function of radial position. The envelopes obtained for different forcing frequencies and jet Weber numbers are then compared to the analytical predictions based on the aforementioned linear stability theories. For the investigated cases present measurement results agree better with aerodynamic stability theory than with predictions based on the prescribed perturbation analysis.

Keywords
Liquid sheet; axisymmetric; modulation; chromatic confocal; capillary waves; linear theory

Introduction
The investigation of radially expanding free liquid sheets has been documented since the early observations made by Savart 1833 [13]. More than a century later Taylor [15] was the first to investigate these films theoretically building on the results of Squire's analysis for planar films [14]. Taylor presented results for sinusoidal film distortion caused by the interaction of the liquid film with the surrounding gas phase due to an initial film disturbance. He included the decrease of film thickness with increasing radius while assuming that this decrease can be neglected locally, i.e. over the distance of one disturbance wavelength. A wider range of experimental investigations on radially expanding liquid sheets generated by the impingement of two head-on impinging jets were undertaken by Huang [5] who described different break-up regimes depending on the jet Weber number $We_d = \rho_l d_j u_j^2/\sigma$ where ρ_l denotes the liquid density, d_j and u_j are the jet diameter and velocity respectively, and σ is the surface tension at the gas-liquid interface. In *Break-Up Regime I* ($100 \leq We_d \leq 500$) the sheet is stable and moves smoothly with only small disturbances which decay radially. The film terminates at a circular

edge where surface tension forces balance the sheet's inertia forces. For higher Weber numbers ($500 < We_d \leq 2000$) Huang describes a *Transition Regime* where up to a Weber number of 800 a cusp-shaped edge of the film is observed, where drops are formed at the peak of the cusp. The shape of the cusp edges is shown to coincide with cardioid wave lines being created by local disturbances on the expanding sheet as already observed by Taylor. For increasing Weber numbers the cusp shaped edges disappear and sinuous waves become dominant, not only for setting the edge of the sheet. In the *Break-Up Regime II* ($2000 < We_d$) the amplitude of the sinuous waves increases with the sheet flapping in a flag-like motion. Following the prescribed classification Clanet and Villermaux investigated smooth liquid sheets ($We_d < 1000$) [3] and flapping liquid sheets ($We_d > 1000$) [18]. In their experimental setup radially expanding sheets were generated by a single liquid jet impinging on the flat end of a cylinder, as previously already considered by Taylor [15]. Based on a local force balance, Villermaux and Clanet [18] arrived at a dispersion relation for both varicose and sinuous disturbances, latter of which matches that of Squire [14] based on local sheet properties. They showed that the sinuous mode of sheet distortion is dominant over the varicose mode for a liquid sheet in a surrounding gas phase. Based on the dispersion relation for a liquid sheet having locally parallel interfaces, while employing the local value of sheet thickness according to the base flow of the radially thinning sheet, their spatial growth analysis provided an amplification factor G_m for the most amplified wave number (see their eq. (18)). The same dispersion relation was also obtained by Lin and Jiang [7] from a a linear stability analysis for a radially expanding inviscid liquid sheet in a surrounding gas phase produced by two coaxial head-on impinging jets. They showed that the appearance of instability depends on the local Weber number $We_H = (\rho_l h u^2)/2\sigma$ based on local sheet thickness h or rather half-thickness $h/2$ and the local film velocity u. For $We_H = 1$ absolute instability is observed whereas for $We_H > 1$ convective instability occurs. The former sets the free edge of the sheet in *Break-Up Regime I* (as already observed by Taylor). The latter corresponds to the instability found in *Break-Up Regime II* and is found upstream of the sheet edge. Bremond et al. [1] were the first to investigate radially expanding liquid sheets onto which sinuous modulations are imposed by mechanically oscillating the impingement cylinder. Their inviscid linear stability analysis built on the prior analysis of Villermaux and Clanet [18]. According to their analysis, the modulated sheet is unstable up to a certain radial location beyond which the wavy disturbances will oscillate with constant amplitude. The amplitude envelope of the film for this so called aerodynamically unstable forcing (*AUF*) can be calculated by Eq.(3.28) in Ref.[1]. For a fixed Weber number a cutoff-frequency f_{co} is found. When the sheet is excited with a frequency higher than this cutoff-frequency the critical radius becomes smaller than the radius of the impingement cylinder rendering the sheet stable w.r.t. the imposed disturbances. This is the so called aerodynamically stable forcing (*ASF*) and the amplitude envelope can be calculated by Eq.(3.22) in Ref.[1]. In contrast, Tirumkudulu and Paramati [16] presented an inviscid linear stability analysis which neglects the influence of the surrounding gas phase. It is based on a regular perturbation expansion of the two-dimensional flow problem in terms of the disturbance parameter $\epsilon = H/R$ with H denoting the sheet thickness at a selected radius R. Their governing equation for sinuous mode disturbances is given by

$$\frac{1}{We_H}\left[\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\frac{\partial \bar{F}_+}{\partial \bar{r}}\right)\right] = \frac{1}{2\bar{r}}\left(\frac{\partial^2 \bar{F}_+}{\partial \bar{t}^2} + 2\frac{\partial^2 \bar{F}_+}{\partial \bar{r}\partial \bar{t}} + \frac{\partial^2 \bar{F}_+}{\partial \bar{r}^2}\right) - \frac{1}{2\bar{r}^2}\left(\frac{\partial \bar{F}_+}{\partial \bar{r}} + \frac{\partial \bar{F}_+}{\partial \bar{t}}\right) \quad (1)$$

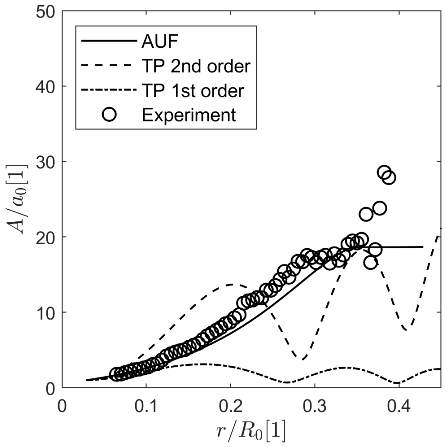
where in contrast to Lin and Jiang [7] $We_H = (\rho_l h u^2)/\sigma$ with u denoting the constant radial sheet velocity of the base flow. $\bar{F}_+$ is the dimensionless position of the sheet centerline, $\bar{r}$ the dimensionless radius and $\bar{t} = tu/R$ the dimensionless time. Since this equation originates from the expansion up to $O(\epsilon^2)$ its solution is subsequently referred to *TP 2nd order*. If the last term on the right hand side ($1/\left(2\bar{r}^2\right)\cdot(...)$) is neglected the equation matches the governing equation obtained from the expansion up to $O(\epsilon)$ (subsequently referred to *TP 1st order*). We note that in the above equation, the left-hand-side only contains terms due to the action of surface tension, and the right-hand-side solely liquid inertia terms, whereby the respective $O(\epsilon^2)$ terms result from the non-zero profile of transverse velocity across the sheet in the base flow (a direct result of sheet thinning) and a deficit in transverse momentum in the disturbed case. While according to Ref.[1] the growth of the sinuous-mode disturbance along the radially expanding sheet is caused by the interaction of the sheet with the surrounding gas phase, Tirumkudulu and Paramati [16] argue that the experimentally observed sinuous-mode instability results merely from a higher-order inertia effect as a consequence of the decrease in sheet thickness in radial direction. Paramati et al. [12] compared the growth rates resulting from both theories and defined two regions in $We - \tilde{\omega}_0$ space for sheets which are harmonically forced at a non-dimensional angular frequency $\tilde{\omega}_0 = \omega_0 d_j / u$. One region where the growth rate according to the theory of Tirumkudulu and Paramati [16] is larger than the one according to the theory by Bremond et al. [1] along the entire sheet and a second region where at at least at one radial section of the sheet the growth rate according to Ref.[1] exceeds the one predicted by Ref.[16].

Experimental Setup

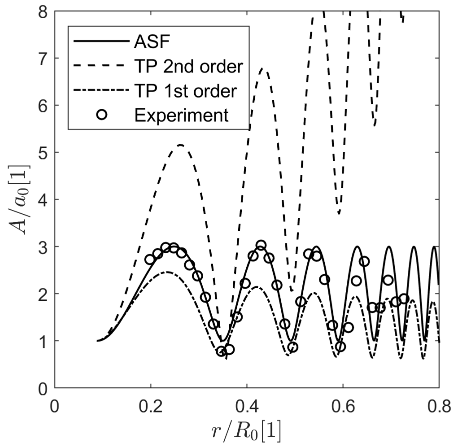
The experimental set-up and flow configuration considered for the present study of radially expanding harmonically modulated free liquid sheets has been used by others; see for example Refs. [15], [1] and [10]. The liquid sheet is generated by a single jet (diameter $d_j = 3\ mm$) impinging vertically onto the flat end of a circular cylinder. As shown by Clanet and Villermaux [3] a loss factor due to friction acting at the impingement surface has to be considered in order to compare experimental and analytical results of the emanating radially expanding free liquid sheet. Following Bremond et al. [1] we choose this loss factor $u/u_j = 0.97$. To avoid a bell like shape of the free liquid sheet at low Weber numbers, a symmetric conical recess was cut into the impingement surface having an inclination angle of 4 ° w.r.t. the horizontal radial direction at its edge. The cylinder is mounted on a shakersystem allowing for vertical oscillatory motion at a defined frequency $f_0 = 2\pi/\omega_0$ and amplitude a_0. The displacement of the sheet in the transverse direction is measured by a Chromatic Confocal Sensor (CCS) System (Precitec CHRocodile 2 DPS controller) with an optical sensor having a measuring range of 1.2 mm and a surface angle tolerance (i.e., maximum allowed angle between measurement direction and normal direction of the measured surface/interface) of $\pm 30°$. The system measures the distance of the (upper) liquid gas interface over time at a frequency of 4 kHz. From this time-resolved signal the amplitude envelope of the undulating sheet can be determined at different radial positions. In the raw signal low-frequency fluctuations are observed at large radii which are likely caused by a non-ideal experimental setup (e.g. air trapped in the inflow of the nozzle). Therefore, the signal is filtered using a highpass filter before determining the amplitude values of the sheet envelope.

Results and Discussion

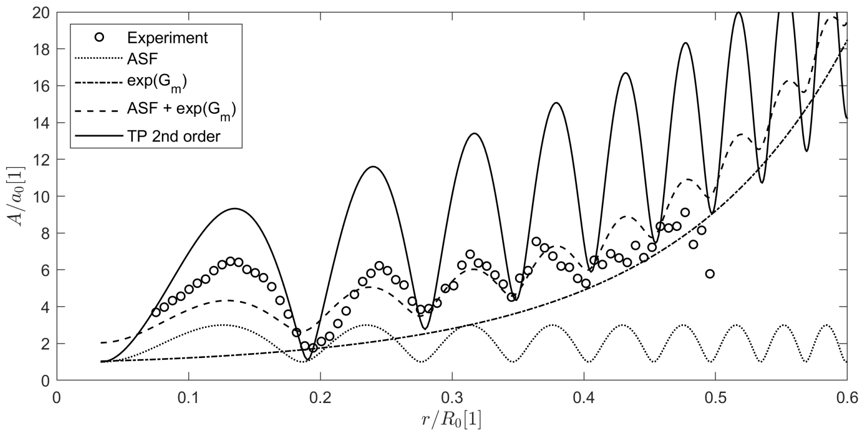
In this section results from our CCS measurements are presented and compared to predictions from the two linear theories described earlier. Fig.1 (a) - (c) and Fig. 2 show the amplitude envelopes of the undulating liquid sheet as a function of dimensionless radius based on the present CCS measurements together with predictions from the linear theories by Bremond et al. [1] and Tirumkudulu and Paramati [16] for different Weber numbers and forcing frequencies. The radius is rendered dimensionless with the theoretical maximum edge radius of the undisturbed sheet $R_0 = We_d/16 \cdot d_j$. Since the forcing frequency ($f_0 = 120\ Hz$) is lower than the cut-off frequency ($f_{co}(We_d = 990) = 293\ Hz$) the operating conditions for the data shown in Fig.1 (a) fall onto the unstable branch of aerodynamic forcing according (AUF) to Ref.[1]. Fig.1 (a) illustrates that the experimental data agree well with the predictions for the AUF condition. In contrast the calculations based on the linear theory according to Ref.[16] predict several maxima and minima which are not observed in the experimental measurements. We also note, a dominant contribution of the $O(\epsilon^2)$ terms in the respective governing equation Eq.1, leading to continuous amplitude growth in the radial direction. For the case presented in Fig.1 (b) the operating conditions fall into an area in $We - \tilde{\omega}_0$ space which is dominated by the thinning of



(a) $We = 990, f_0 = 120\ Hz, a_0 = 15\ \mu m$



(b) $We = 323, f_0 = 150\ Hz, a_0 = 30\ \mu m$



(c) $We = 850, f_0 = 260\ Hz, a_0 = 10\ \mu m$

Figure 1. Amplitude envelope of undulating sinusoidally forced liquid sheet as function of radial position for different forcing frequencies f_0 and Weber numbers We_d. Experimental data from CCS measurements (circles) compared to linear theories by Bremond et al. [1] (AUF,ASF) and Tirumkudulu and Paramati [16] (TP 1st, 2nd order). Cases: (a) AUF with growth rates of both theories being similar; (b,c) ASF with growth rates based on Ref.[16], i.e., due to higher-order inertia effects as a consequence of film-thinning, being dominant.

the expanding sheet according to Ref.[12] and which pertains to an area of ASF according to Ref.[1]. Both theories predict several nodes and anti-nodes in the sheet envelope which match with experimental data. However, in stark contrast to the prediction based on the linear theory of Ref.[16] the amplitude envelope obtained from the experimental data does not increase with radius. If the second order terms are neglected in the respective governing equation Eq.1 the calculated results agree better with the experimental data, even though now the amplitude envelope is underpredicted and also decreases significantly stronger in radial direction than observed in the measurements. However, comparing the experimental data with the calculation for the ASF condition according to the linear theory of Ref.[1], we find excellent agreement. The results shown in Fig.1 (c) are obtained at operating conditions for which according to Ref.[12], sheet distortion is dominated by the unstable growth rate predicted by the linear theory of Ref.[16]. For this operating point experimental data and predictions from linear theory according to Ref.[16] agree with respect to the location of nodes and anti-nodes and with respect to the amplitude values of the envelope nodes (i.e. local minima of the sheet envelope). However, the amplitude values of the anti-nodes, i.e. local maxima of the sheet envelope are found to be significantly smaller in the experiments than predicted by the prescribed linear theory. Also, while linear theory predicts a virtually constant amplitude difference between neighboring nodes and anti-nodes, experimental measurements shows a continuous decrease of the anti-node amplitudes approaching those of the neighboring nodes. Based on linear theory of Ref.[1]

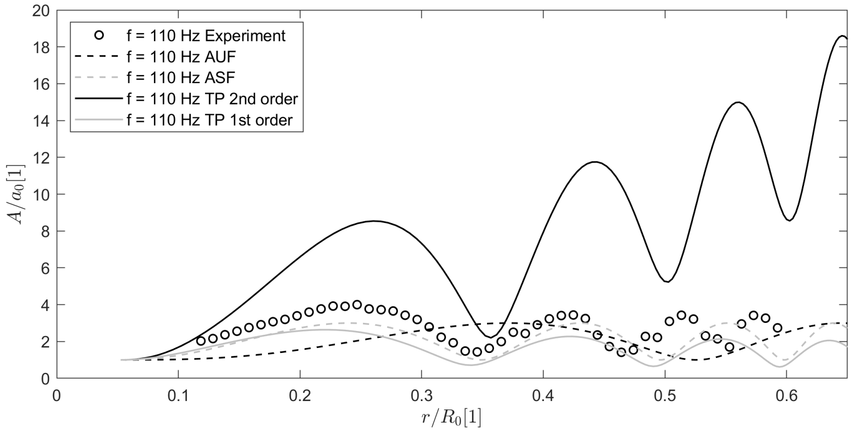


Figure 2. Amplitude envelope of undulating sinusoidally forced liquid sheet as function of radial position for $f_0 = 110Hz$, Weber number $We_d = 540$ and forcing amplitude $a_0 = 40\mu m$. Experimental data from CCS measurements (circles) compared to linear theories by Bremond et al. [1] (AUF,ASF) and Tirumkudulu and Paramati [16] (TP 1st, 2nd order). Case: AUF with growth rates based on Ref.[16], i.e., due to higher-order inertia effects as a consequence of film-thinning, being dominant.

the case illustrated in Fig.1 (c) corresponds to an ASF condition. Accordingly, the respective analytical solution for the amplitude envelope does not increase amplitude envelope along the radius at all, even though the predicted location of nodes and anti-nodes matches the experimental results. A possible explanation for this discrepancy is that the growth of the amplitude envelope is caused by a superposition of the oscillation caused by the imposed acting forcing of the sheet at frequency f_0 (dotted line in Fig.1(c)) and a disturbance growth based on the naturally most amplified wavenumber (dash-dotted line in Fig.1(c)). This seems a plausible explanation considering the fact that Ref.[5] describes the transition to unstable sheet behavior for $We_d > 800$ based on radially expanding sheets without active modulation. If we simply add the amplitude prediction according to the linear theory of Ref.[1] for the considered ASF condition to the amplitude predicted for the most amplified natural mode in this case, the result matches remarkably well with the experimental results. However, it should be noted here that this would require that the initial disturbance causing the natural growth is of the same magnitude as the external forcing (i. e., $a_0 = 10\ \mu m$).

Fig.2 depicts the normalized amplitude envelope along the normalized radial direction for $We = 540, f_0 = 110\ Hz, a_0 = 40\ \mu m$. Similar to the operating conditions considered in Fig.1 (b) and (c) the growth rate prediction according to Ref.[16] should dominate over the growth rate value according to Ref.[1] in this case, resulting in a considerable increase in amplitude envelope over the radial direction. Clearly this prediction is not supported by the experimental results. It should be noted, however, that excluding the $O(\epsilon^2)$ terms in the respective linear theory and in the corresponding govering equation Eq.1, yields significant better agreement with the experimental results. As illustrated in Fig.2, the behavior of the sheet in terms of its amplitude envelope is best predicted by the linear aerodynamic theory according to Ref.[1] based on ASF condition. However, the forcing frequency ($f_0 = 110\ Hz$) is actually slightly lower than the cut-off frequency($f_{co} = 113\ Hz$). Therefore the investigated condition actually corresponds to an AUF condition, which predicts the same magnitude of the amplitude envelope, but different radial positions for nodes and anti-nodes. The observation that the prediction based on an ASF condition agrees better with the experimental results than that based on AUF condition, can be explained by the fact that the forcing frequency is very close to the cut-off frequency; such that a slight reduction of the Weber number (e.g. as a consequence of energy loss due to friction at the impingement cylinder) would lead to a transition from an AUF to an ASF area in the $We - \tilde{\omega}_0$ space.

Conclusions

Radially expanding free liquid sheets subject to sinusoidal modulations were investigated using chromatic confocal measurements and the results were compared to two different existing linear theories for this flow geometry. One which predicts growth of forced sinuous disturbances as a consequence of aerodynamic interactions between the liquid sheet and its surrounding gas phase; and a second one which predicts sinuous-mode instability of the expanding film due to higher-order inertia effects as a consequence of film-thinning and in the absence of a surrounding gas phase. We find that, even for cases where the growth rate predicted by the latter theory is larger at all radial positions than that of the prescribed aerodynamic wave-growth theory, results from the aerodynamic theory agree better with our experimental data. Only for one investigated case ($We = 850,\ f_0 = 260\ Hz$) the experimental results do not match with theoretical predictions. However, here the growth of the naturally most amplified wave number might play a relevant role.

Nomenclature

a_0	forcing amplitude [m]	$\bar{t}$	dimensionless time [1]
d_j	jet diameter [m]	u_j	jet velocity [m s^{-1}]
f_0	forcing frequency [Hz]	u	sheet velocity [m s^{-1}]
$\bar{F}_+$	dimensionless sheet center line [1]	We_d	jet Weber number [1]
h	sheet thickness [m]	We_H	local Weber number [1]
H	characteristic length scale in axial direction [m]	ϵ	disturbance parameter [1]
$\bar{r}$	dimensionless radius [1]	ρ_l	liquid density [kg m^{-3}]
R	characteristic length scale in radial direction [m]	σ	surface tension [kg s$^{-}1$ m^{-1}]
R_0	maximum radius of the undisturbed sheet [m]	ω_0	forcing angular velocity [Hz]
t	time [s]	$\tilde{\omega}_0$	dimensionless forcing frequency [1]

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